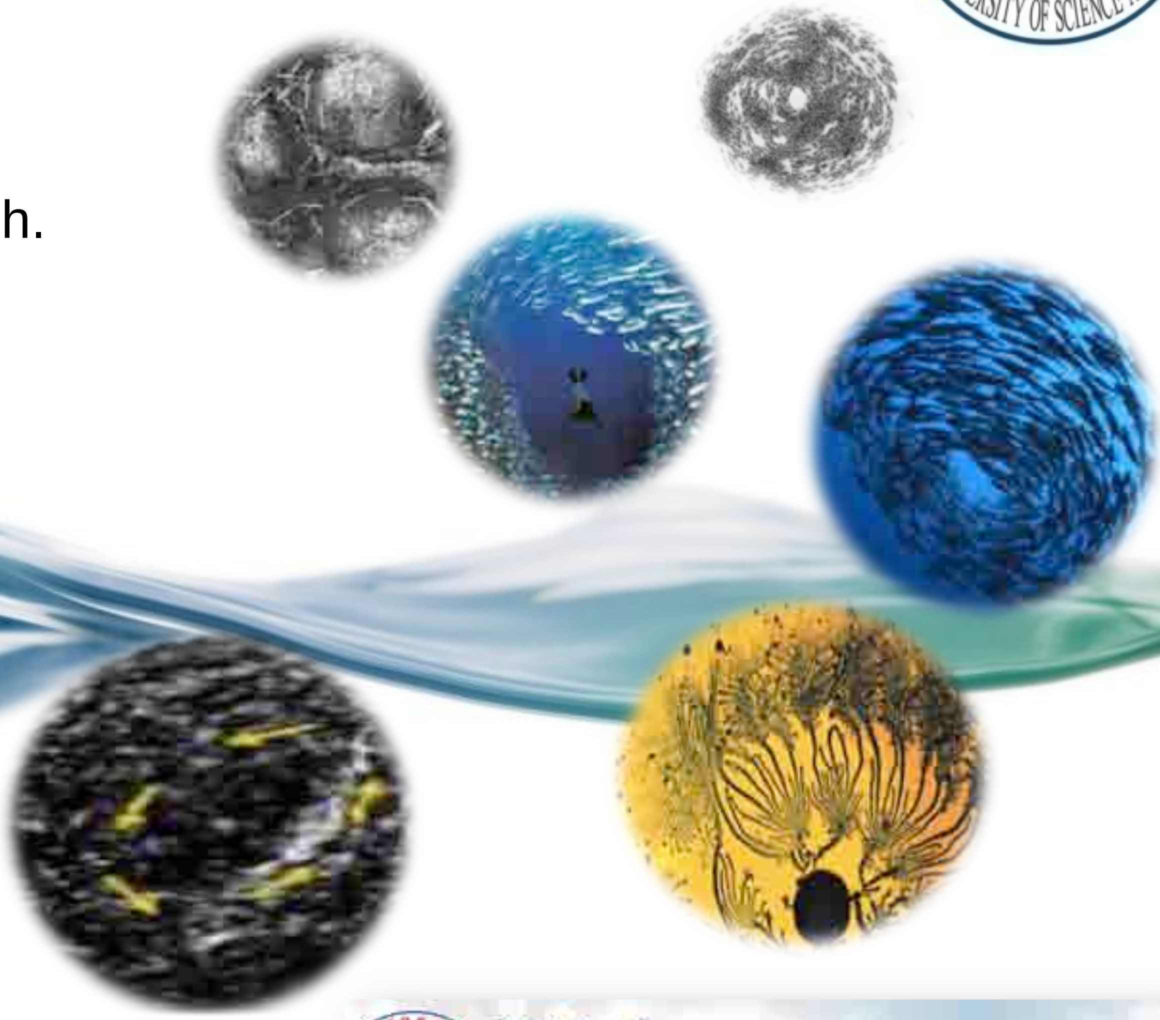
Collective torus behavior analysis and control

Hai-Tao Zhang
Dept. Control Sci. & Eng.
Huazhong Univ. Sci. & Tech.



May 10, 2012

Outline

- Background
- Controller design and analysis
- Experiments
- Conclusion

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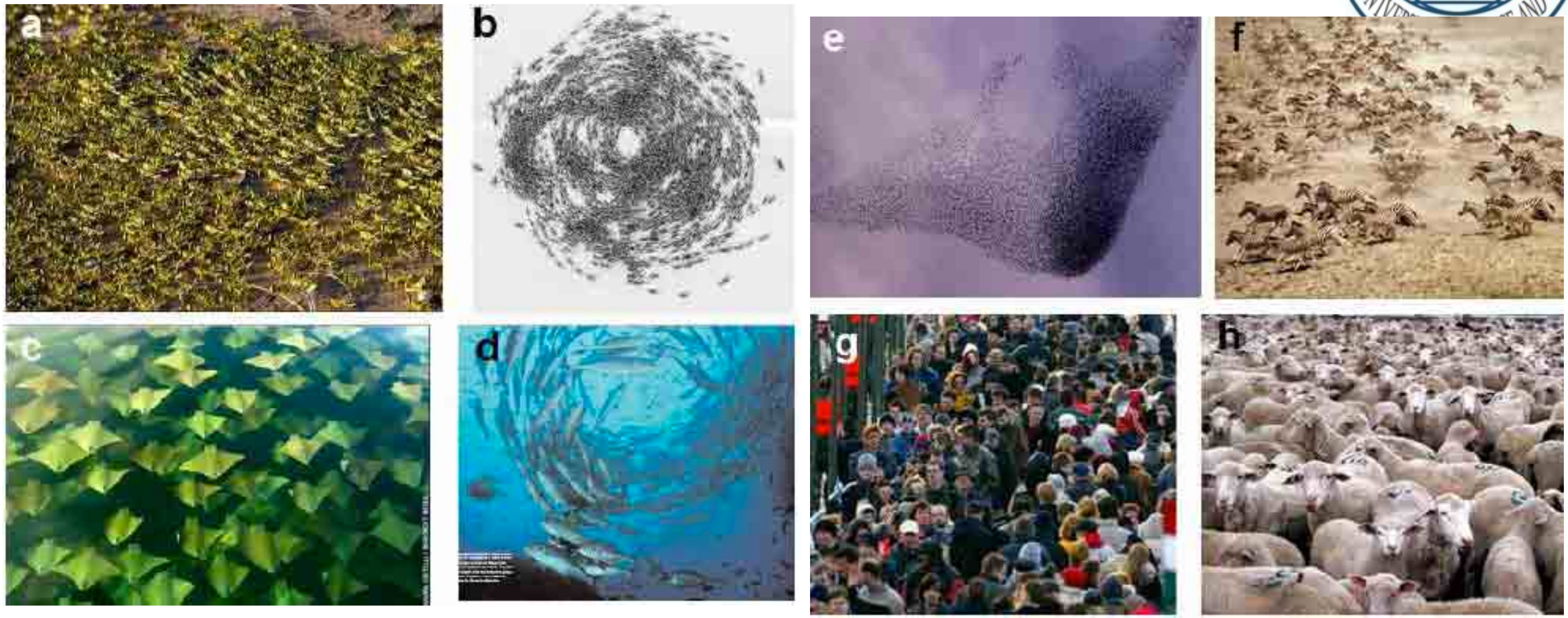
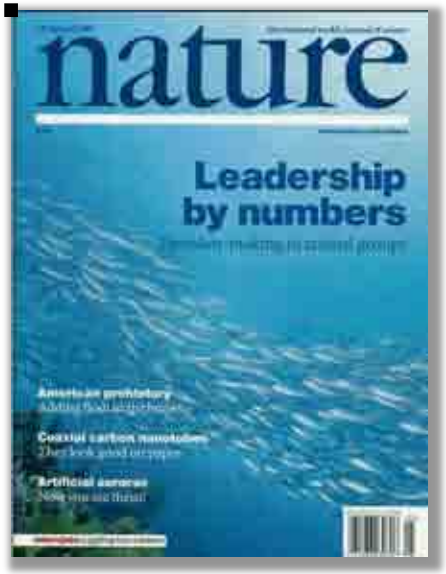
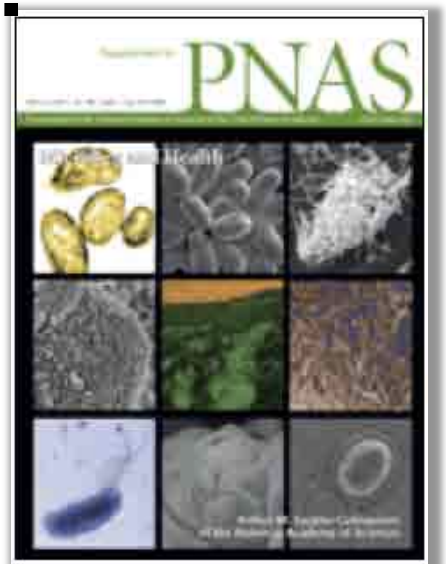
Background

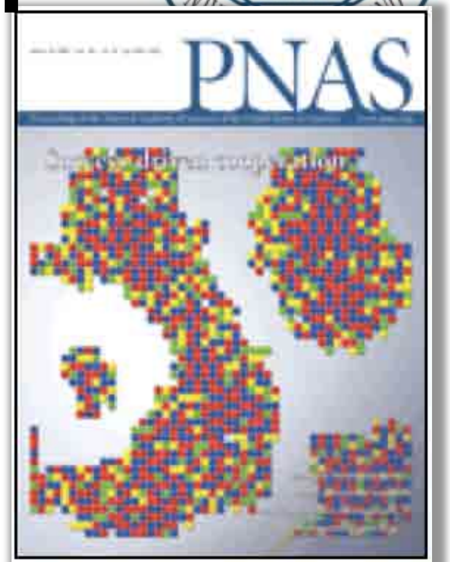


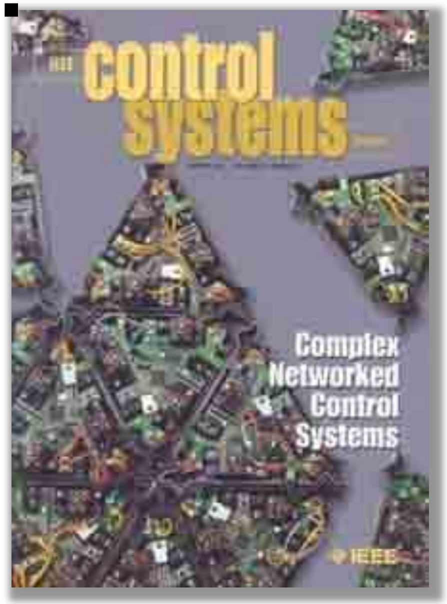
•Collective emergence of natural groups: low level individual intelligence and inter-agent connections create highly-coordinated collective behavior

Top journal reports







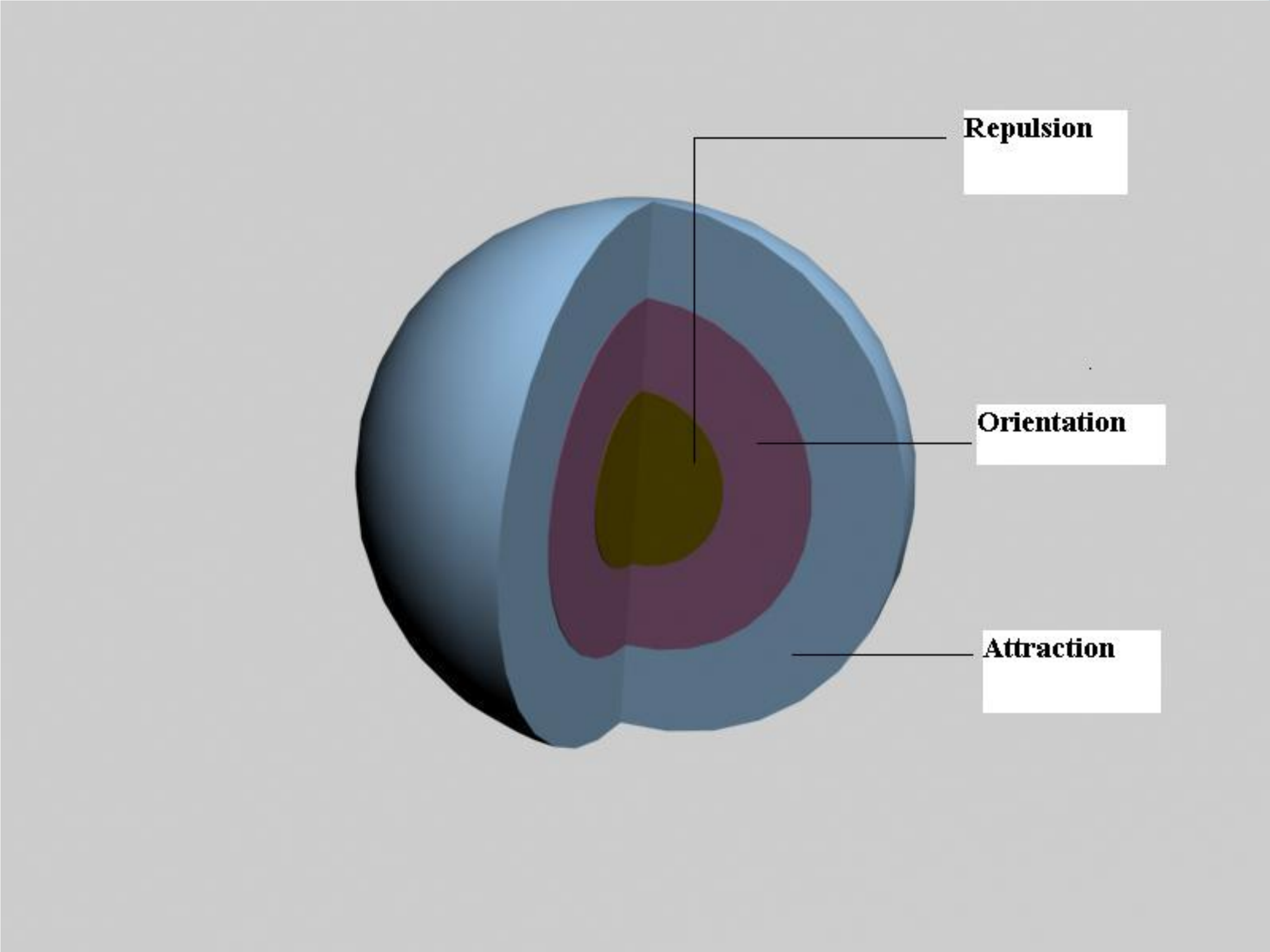


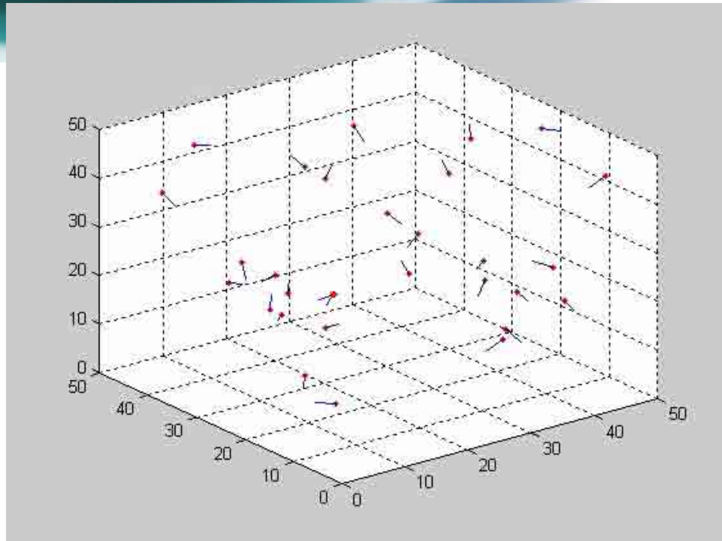
- Top journals have used more and more pages to report the relevant results

- The publication proportion is surging to about 20%in *IEEE TAC* and *Automatica*

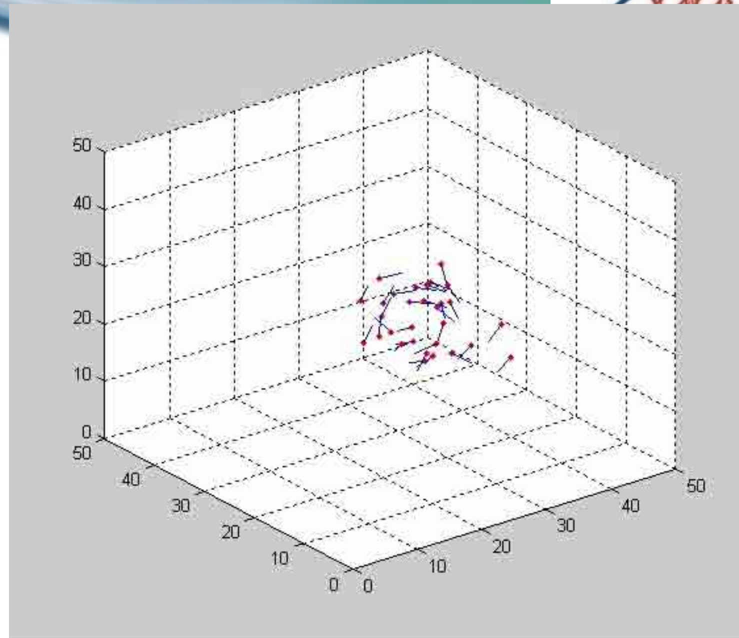
[redacted]

Couzin's Three-Sphere Model
(The New York Times, Nov.10, 2007)

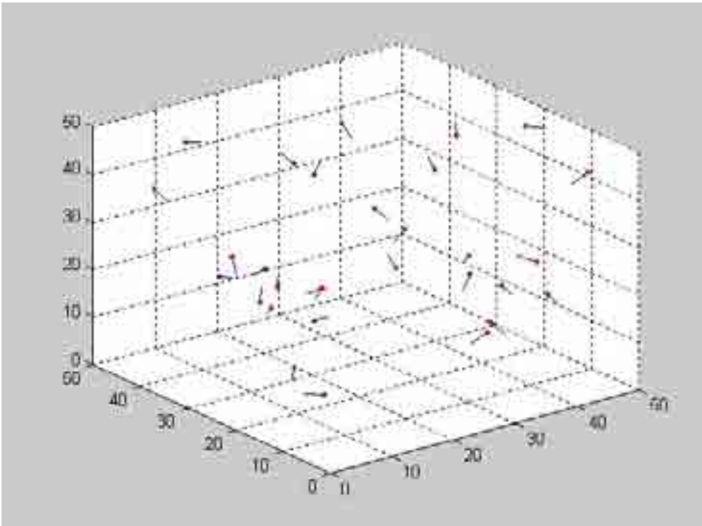




swarm



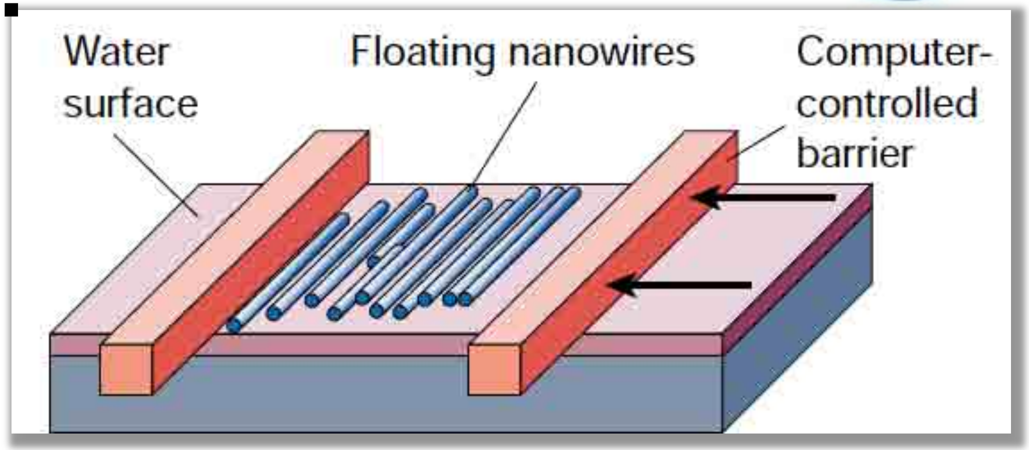
torus

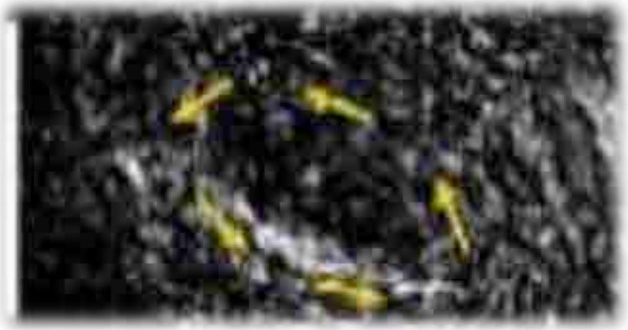


migration

Physical, Chemical and Bio-molecule Torus

Vertically vibrated granular rods, *Physical Review E*, 67: 031303





Bio-molecule actin filament, *Nature,* 467:73–77

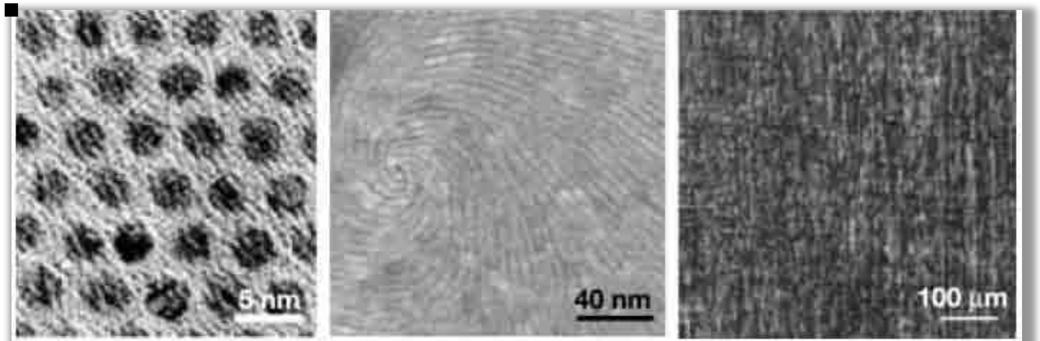


Figure 2 Langmuir–Blodgett assembly: (left to right) dots[9], rods[10] and wires[1].

Nature 425,243

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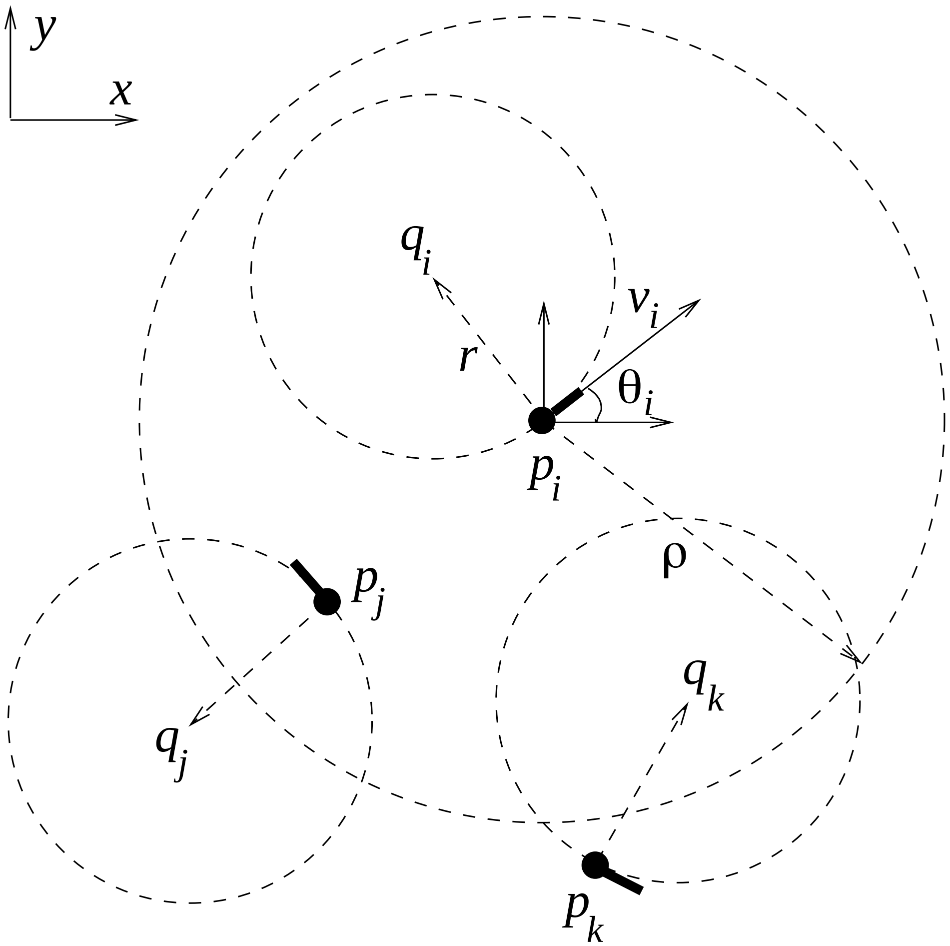
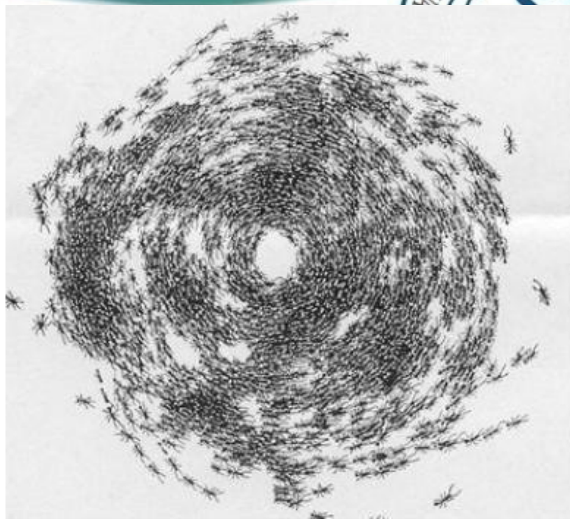
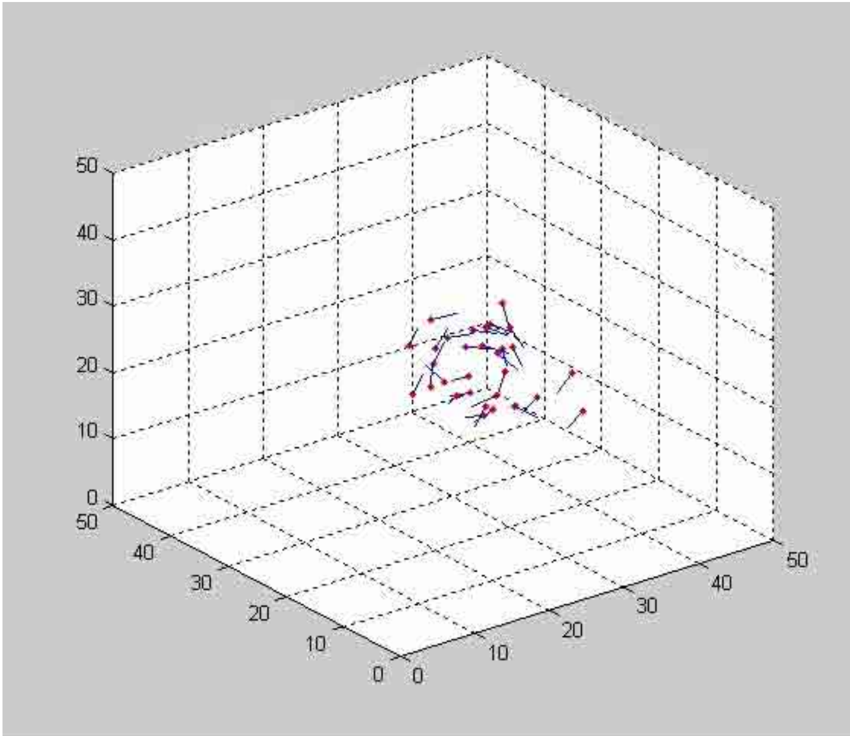
Model





Problem description

- Individuals are indistinguishable.
- No reference beacon exists
- What can each agent perceive?
  - *Neighbors' moving angels*
  - *Relative positions*
- No attraction



Model

- Agents

$$\dot{x}_i = v_i \cos\theta_i, \dot{y}_i = v_i \sin\theta_i, \dot{\theta}_i = \omega_i, i \in \{1, \cdots, n\}$$

- Neighbors

$$N_i := \left\{ j \,\middle|\, \|p_{ij}\| < \rho \right\}, p_i = [x_i, y_i]^T$$

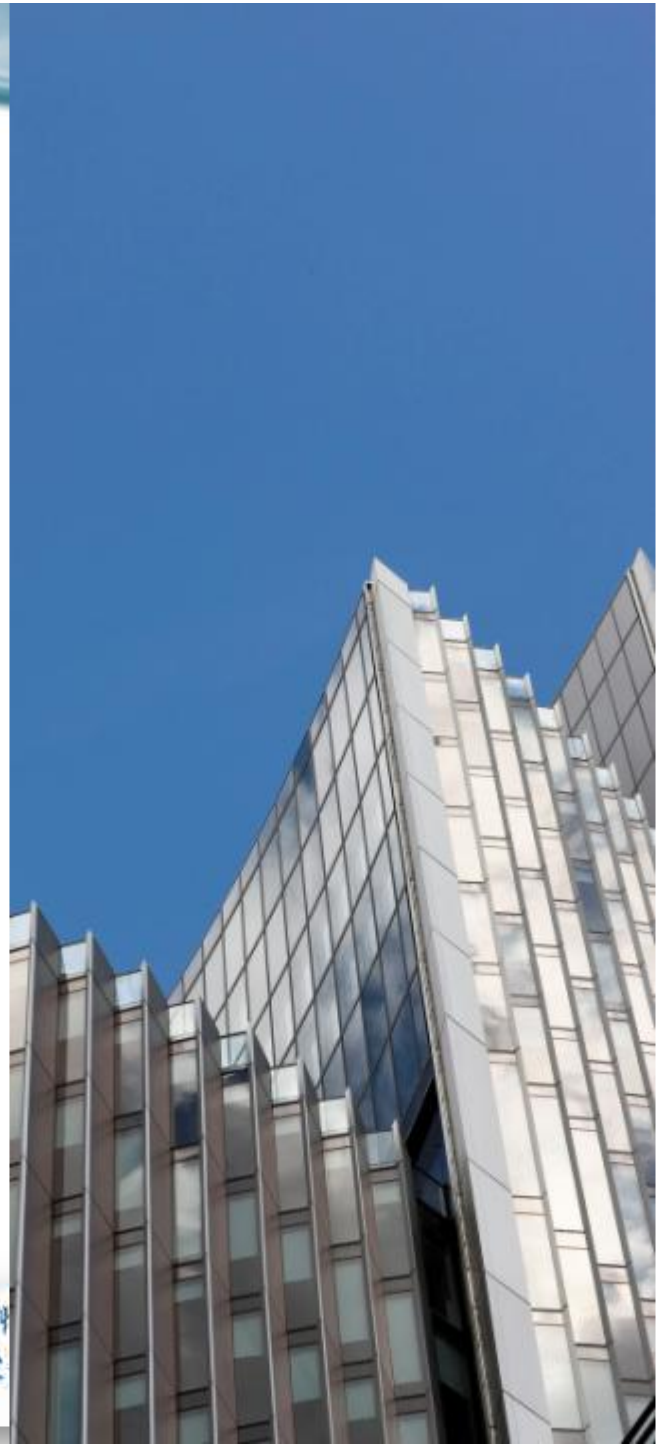
- Control

$$[v_i, \omega_i]^T = \sum_{j \in N_i(p)} \kappa\left(p_{ij}, \theta_i, \theta_j\right)$$

Torus motion

- A trajectory *p(t)* of multi-agent group is called a torus if there exist $T > 0, r > 0,$ and $q_0 = [\xi_0, \zeta_0]^T$ such that

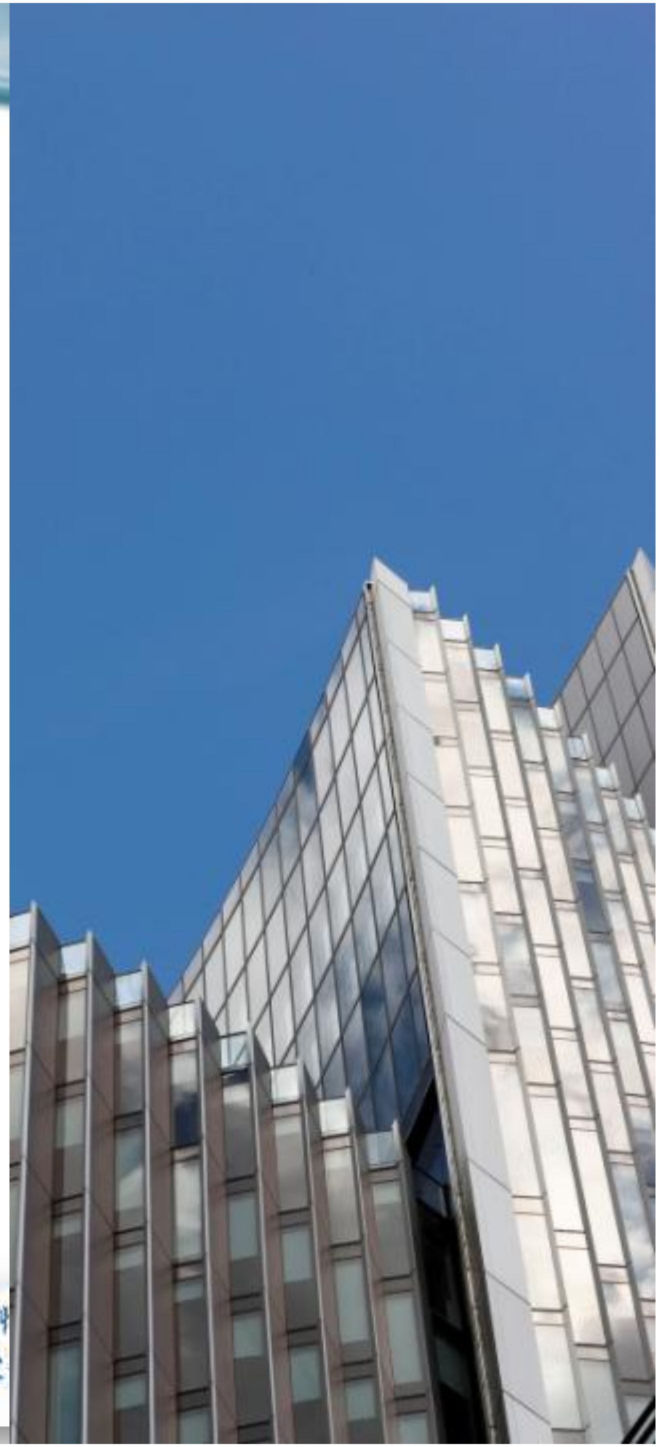
$$p_i(t+T) = p_i(t), \; p_i(t) - q_0 = r\angle\phi_i(t)$$
$$\dot{\phi}_i(t) \geq 0, \forall t \geq 0, \forall i \in \mathbf{N}$$



Objective

- Find a control algorithm for each individual agent such that the trajectory *p(t)* of the closed-loop system converges to a torus motion $\breve{p}(t)$, i.e.,

$$\lim_{t\to\infty}\left(p(t)-\breve{p}(t)\right)=0$$



Main results

- Forward velocity controller (form circular behavior)

$$v_i = r\omega_i - \sum_{j\in \mathrm{N}, j\neq i} \alpha\left(\left\| p_{ij} \right\|\right)\left[\cos\theta_i, \sin\theta_i\right]\begin{bmatrix} x_{ij} - rs_{ij} \\ y_{ij} + rc_{ij} \end{bmatrix}, r = {v_0}/{\omega_0}$$

$$\omega_i = \omega_0, i \in N$$

$$c_{ij} = \cos\theta_i - \cos\theta_j, s_{ij} = \sin\theta_i - \sin\theta_j$$

Main results

- Coordinate transformation (*edge* ->*center*)

$$\Phi:(p_i,\theta_i)\mapsto(q_i,\theta_i)=\begin{cases}\xi_i = x_i - r\sin\theta_i \\ \zeta_i = y_i + r\cos\theta_i\end{cases}$$

$$q_i = [\xi_i,\zeta_i]^T, i\in N$$

Main results

- Closed-loop system

$$\dot{q} = -K(\theta) f(q,\theta), \dot{\theta} = \omega_0 e$$

$$\bar{q}(t) = \frac{1}{T}\int_t^{t+T} q(\tau)d\tau$$

$$\bar{q}(t) - q(t) = O(1/\omega_0), \dot{\bar{q}} = -f(\bar{q},\theta) + O(1/\omega_0)$$

- Average system

$$\dot{q} = -f(q,\theta), \dot{\theta} = \omega_0 e$$

Main results

- *Convergence of torus motion center*

Suppose there exists an infinite time sequence $t_1 < t_2 < t_3 < \cdots$ such that, for any $[t_i, t_{i+1}), i = 1, 2, \cdots,$ the trajectory $p(t)$ is uniformly jointly connected. Then, there exists a collective circular motion $\breve{p}(t)$ such that $p(t)$ asymptotically converges to $\breve{p}(t)$, i.e,

$$\lim_{t\to\infty} \left(p(t) - \breve{p}(t)\right) = 0$$

Main results (*IEEE TAC* 56(2) 2011 and *Automatica*-regular 47(9) 2011)

- Rotational controller (control *phase distribution)*

$$\omega_i = \omega_0 + \sum_{j \in \mathrm{N}, j \neq i} \beta\left(\left\|p_{ij}\right\|\right) \operatorname{sgn}^+\left(\sin\theta_{ij}\right), \quad i \in \mathrm{N}$$

$$\beta(s) = \begin{cases} 0, s \geq \sigma \\ > 0, s < \sigma \end{cases}$$

$\sigma := 2r\sin(\varphi/2) > 0$ where $0 < \varphi \leq 2\pi/n$ is the minimal arc angle between each two agents

Main results

- *Phase distribution*

1) *If* $p_i(t) \neq p_j(t),\ \forall (i,j) \in E$ *holds for* $t = 0$, *then so does it for all* $t \geq 0$.

2) *The limit* $\tilde{p}_{ij} = \lim_{t \to \infty} \left\| p_{ij}(t) \right\|$ *exists and* $\tilde{p}_{ij} \geq \sigma$ *for all* $(i,j) \in \mathrm{E}$. *In particlualr* $\varphi = 2\pi / n$, *then* $p_i(\infty),\ i \in \mathrm{N}$ *are equally loated over the circle.*

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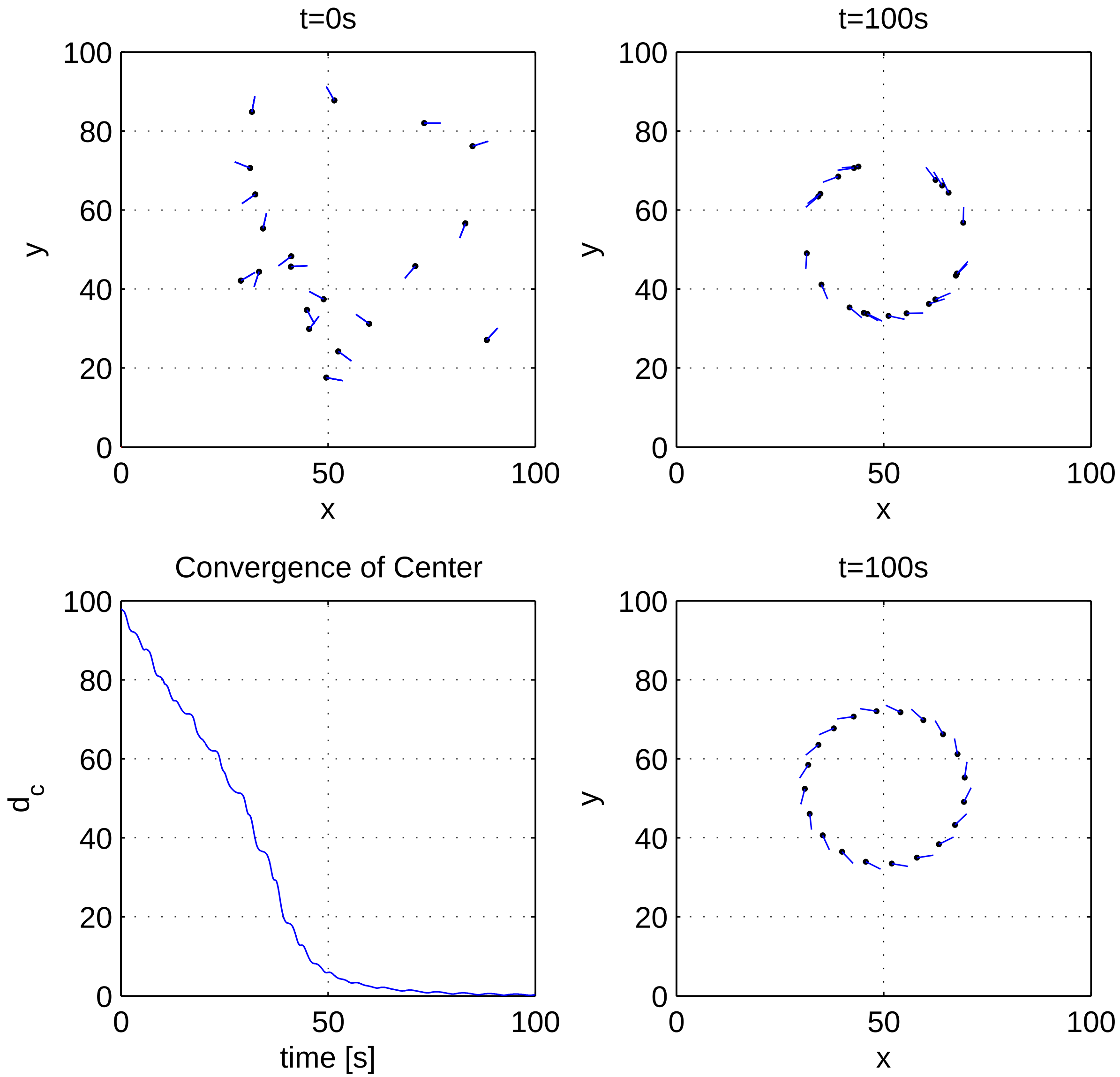
Simulations

- *Case 1:*

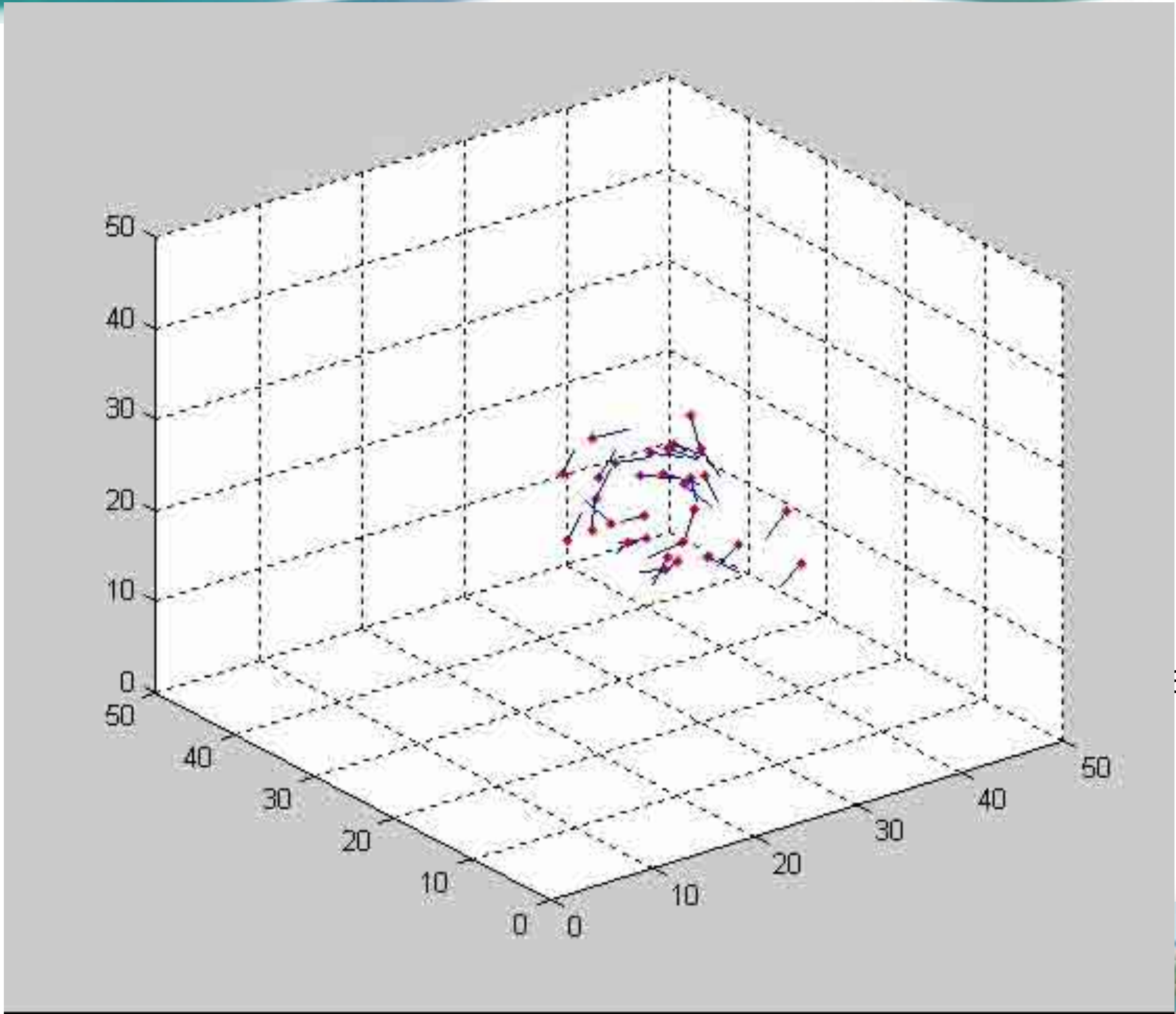
Single layer torus

$$d_c(t) =$$
$$\max\{\|q_{ij}(t)\|, (i,j)\in \mathrm{I}$$



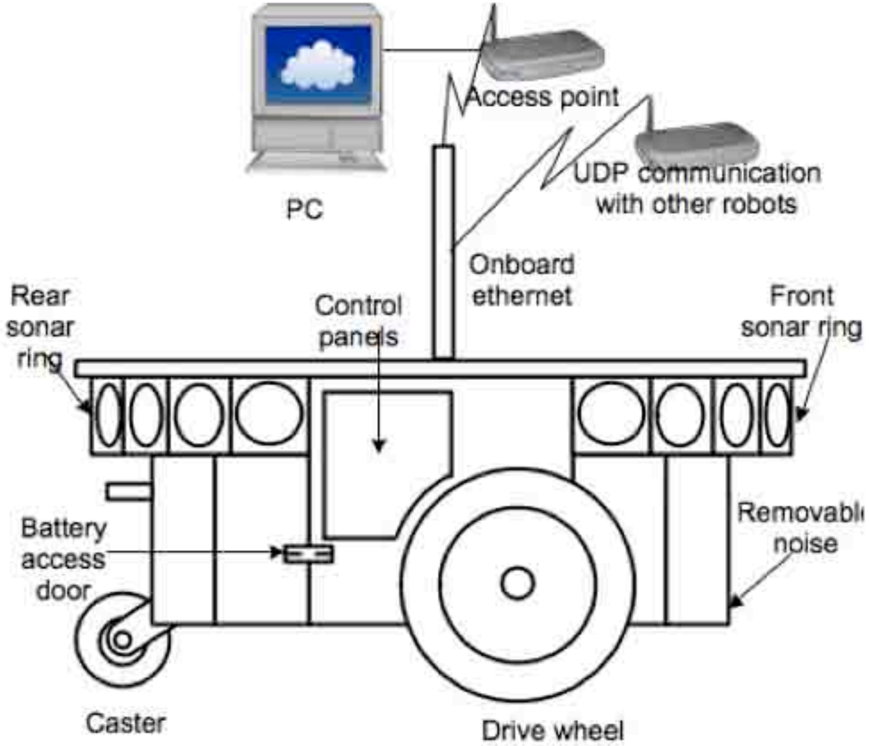
Simulations

Simulations



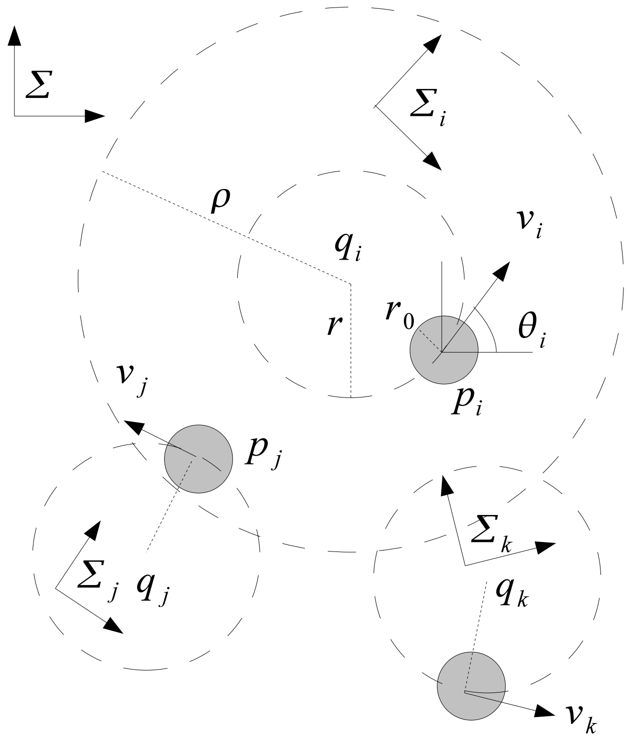
Experiments (*IEEE TCST*, to appear 2012)

- *Multi-robot system*



Experiments

- ***Information exchange***
  - the robot *i*'s own moving direction with respect to its reference frame Σi
  - the direction of the robot *j* observed by the robot *i* using sonar sensors with respect to its reference frame Σi.
  - the distance between the robots *j* and *i* measured by the robot *i* using sonar sensors.
  - Moving angle difference measured by the robot *j* and transmitted to the robot *i* through UDP connection

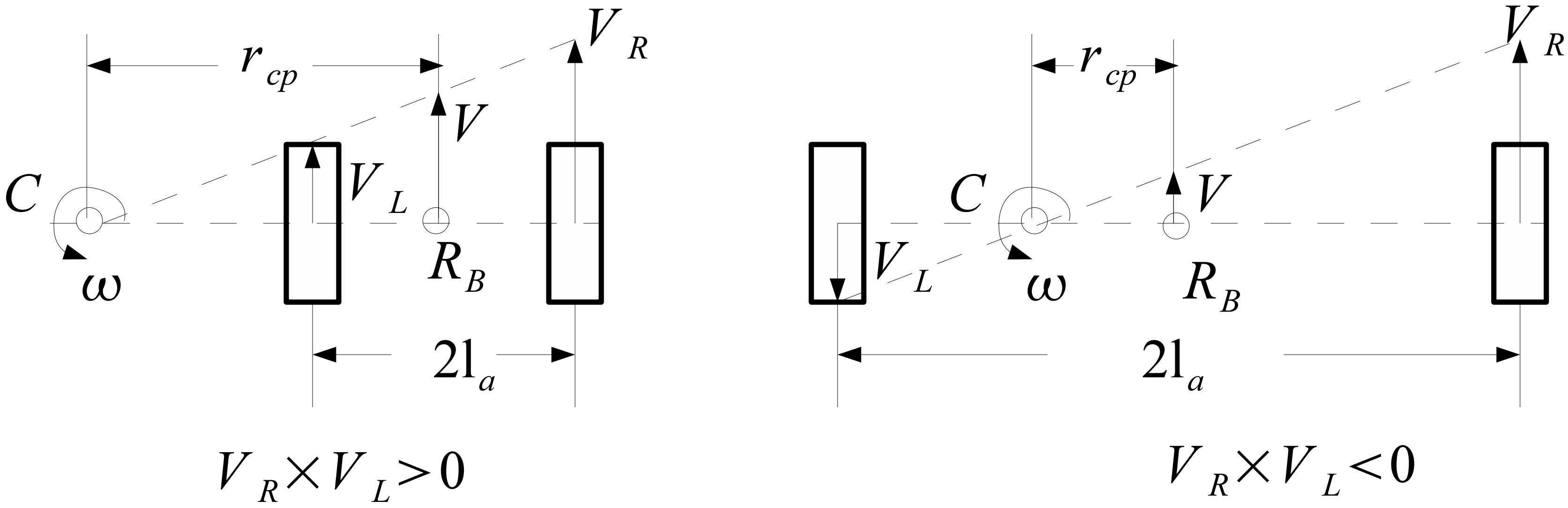


Experiments

- *Real movie*

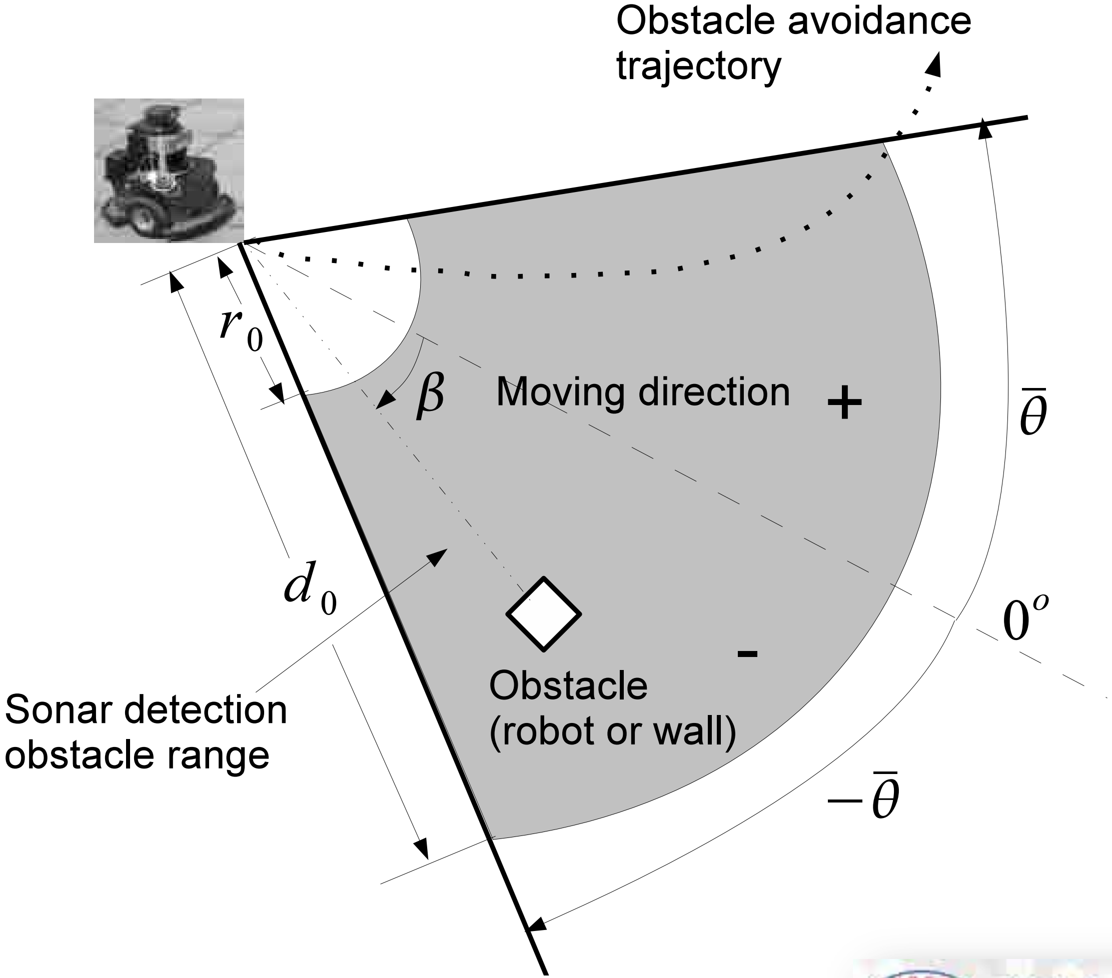
Experiments

- *Practical issue 1: bi-wheeled dynamics*



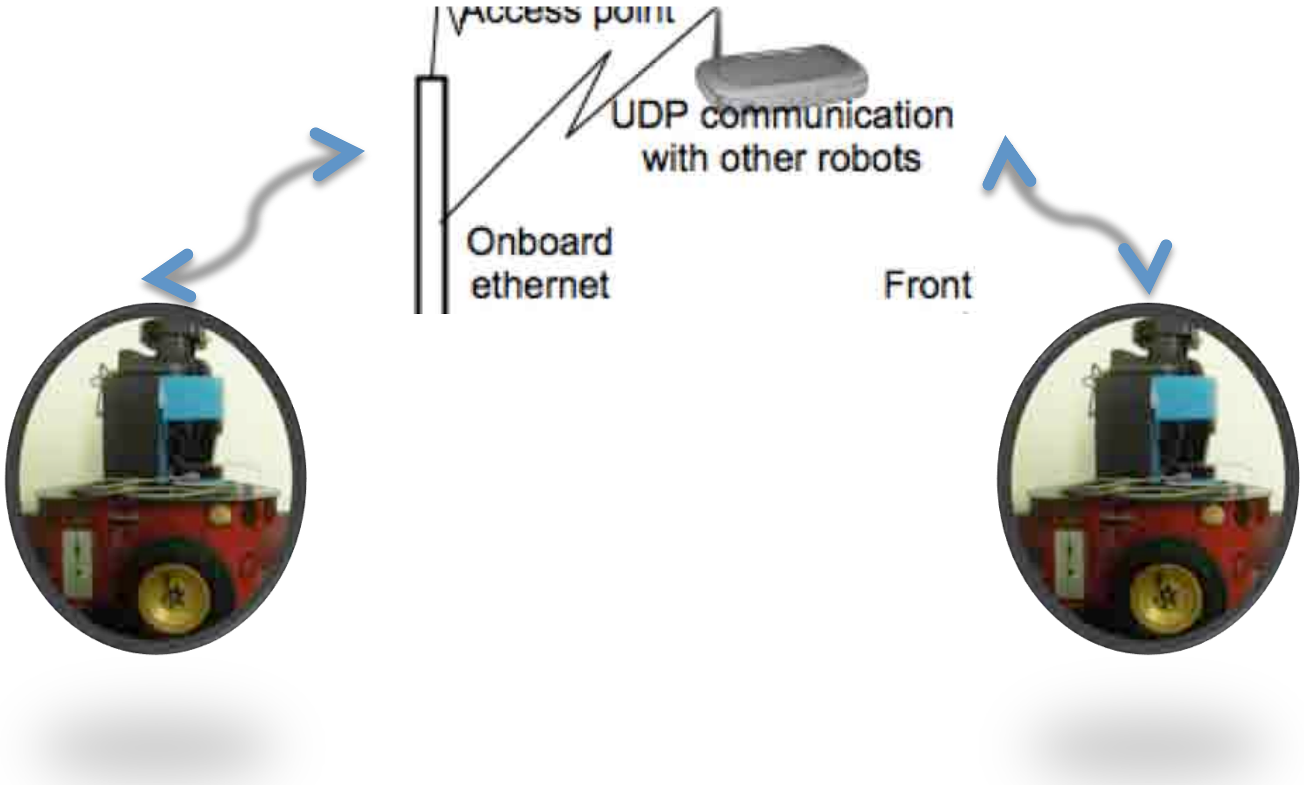
Experiments

- *Practical issue 2: obstacle avoidance*



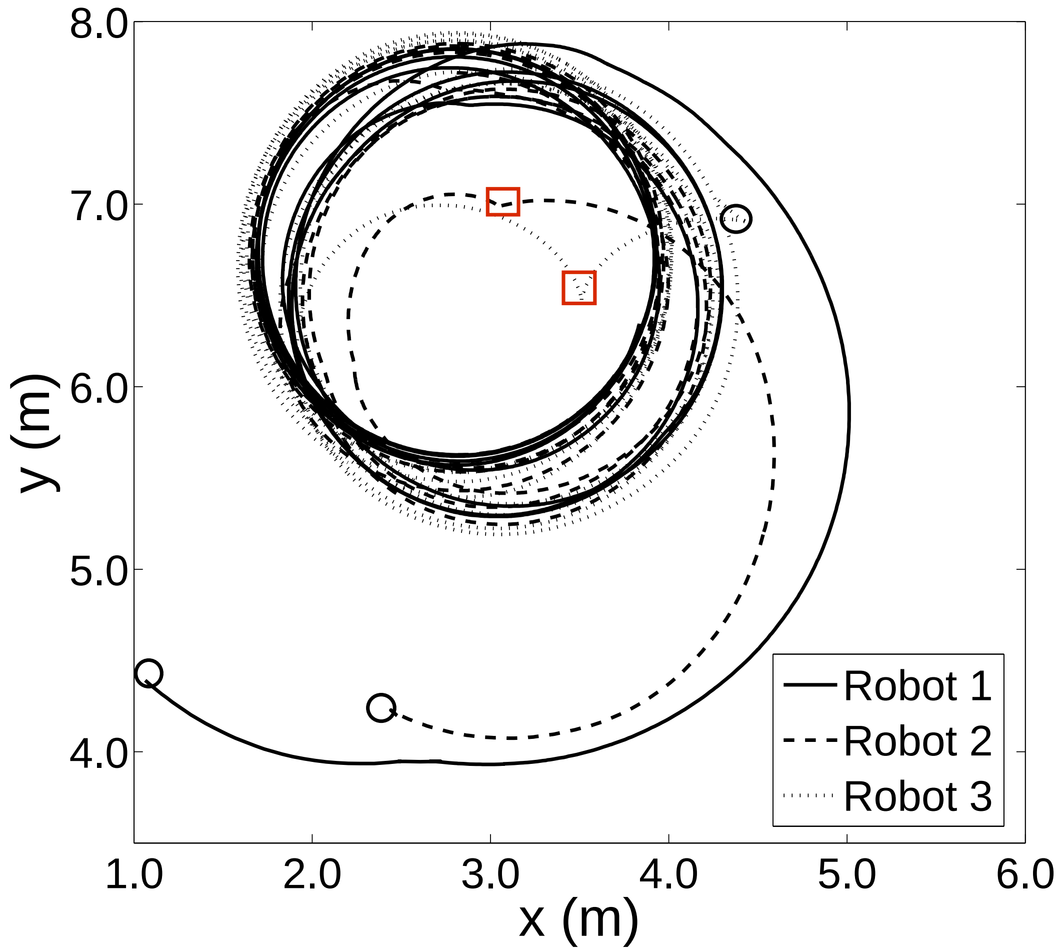
Experiments

- *Practical issue 3: UDP communication loss*



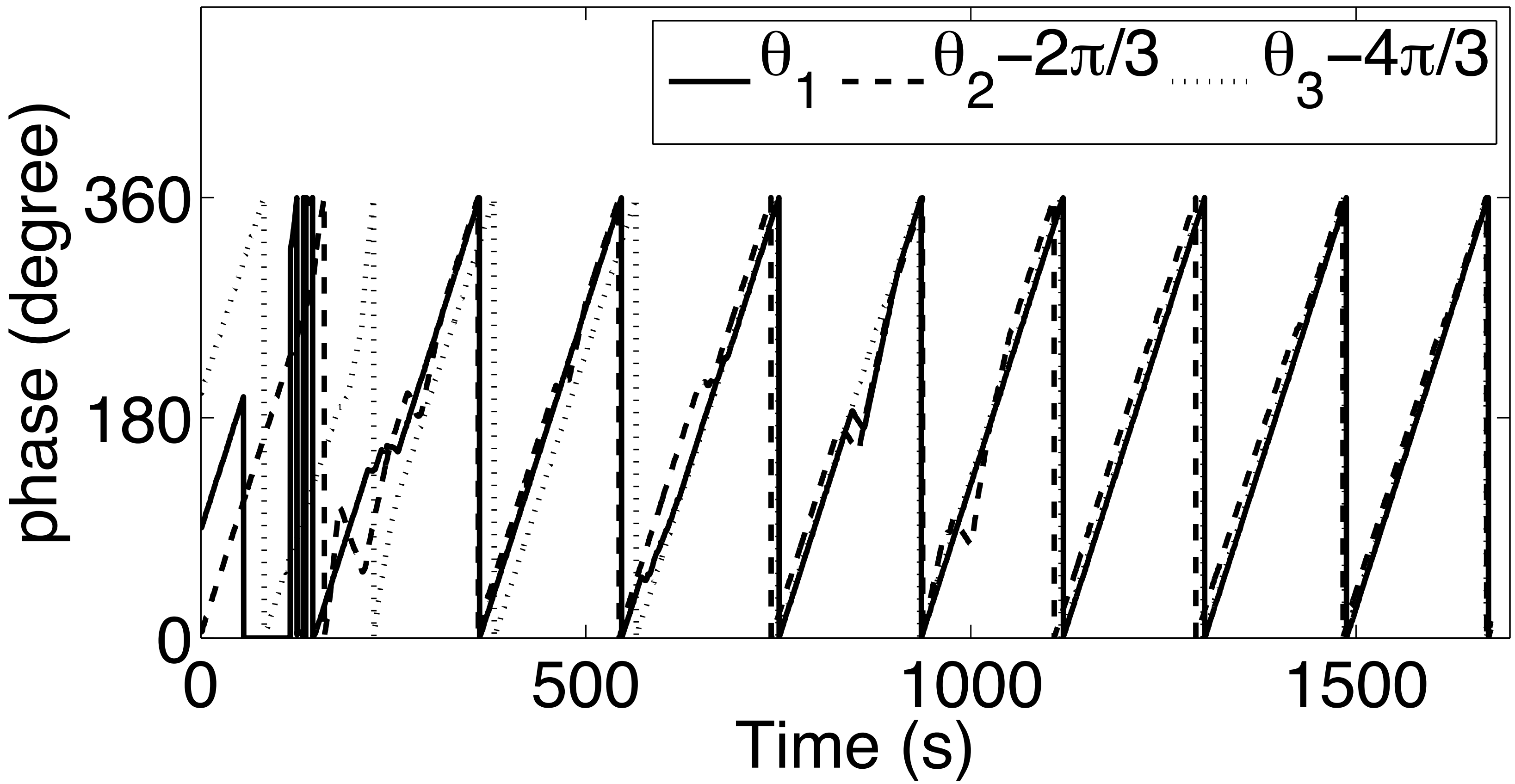
Experiments

- *Moving trajectories*



Experiments

- *Consensus procedures of the centers and moving angles*



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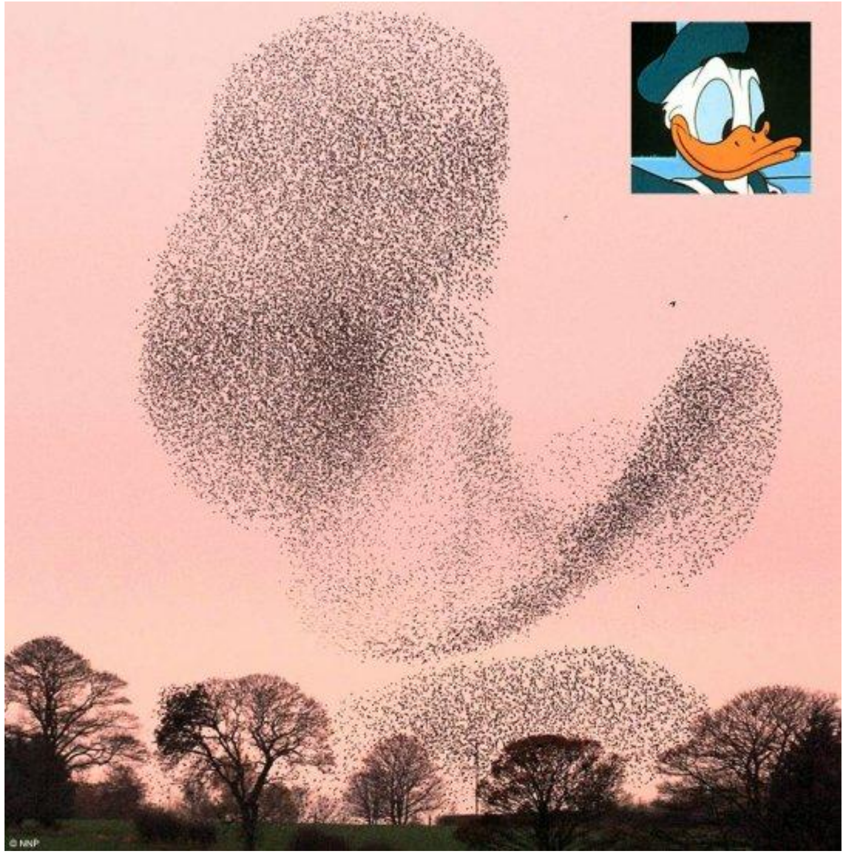
Conclusion

- Collective circular behavior or torus is prevalent in both living and non-living natural groups
- Decentralized control algorithm is designed with leader-free and no-beacon environment
- Collision avoidance, phase distribution, communication loss are considered
- Fulfilled by DX II multi-robot systems

Thank you!



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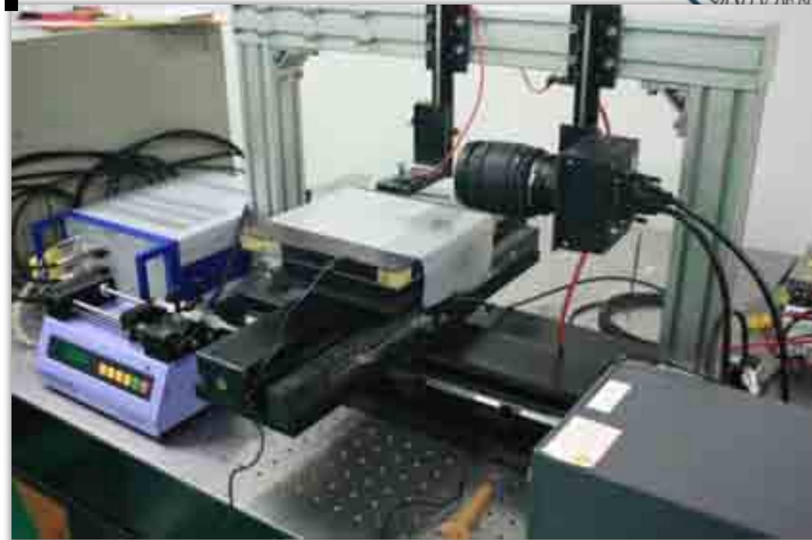
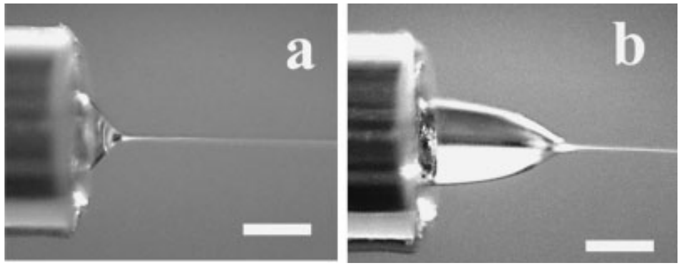
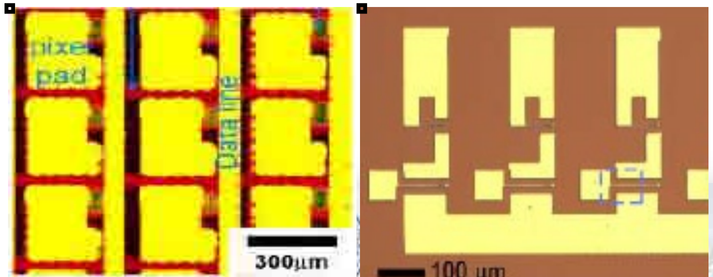
Future applications

•先锋DX-II型多机器人系统;

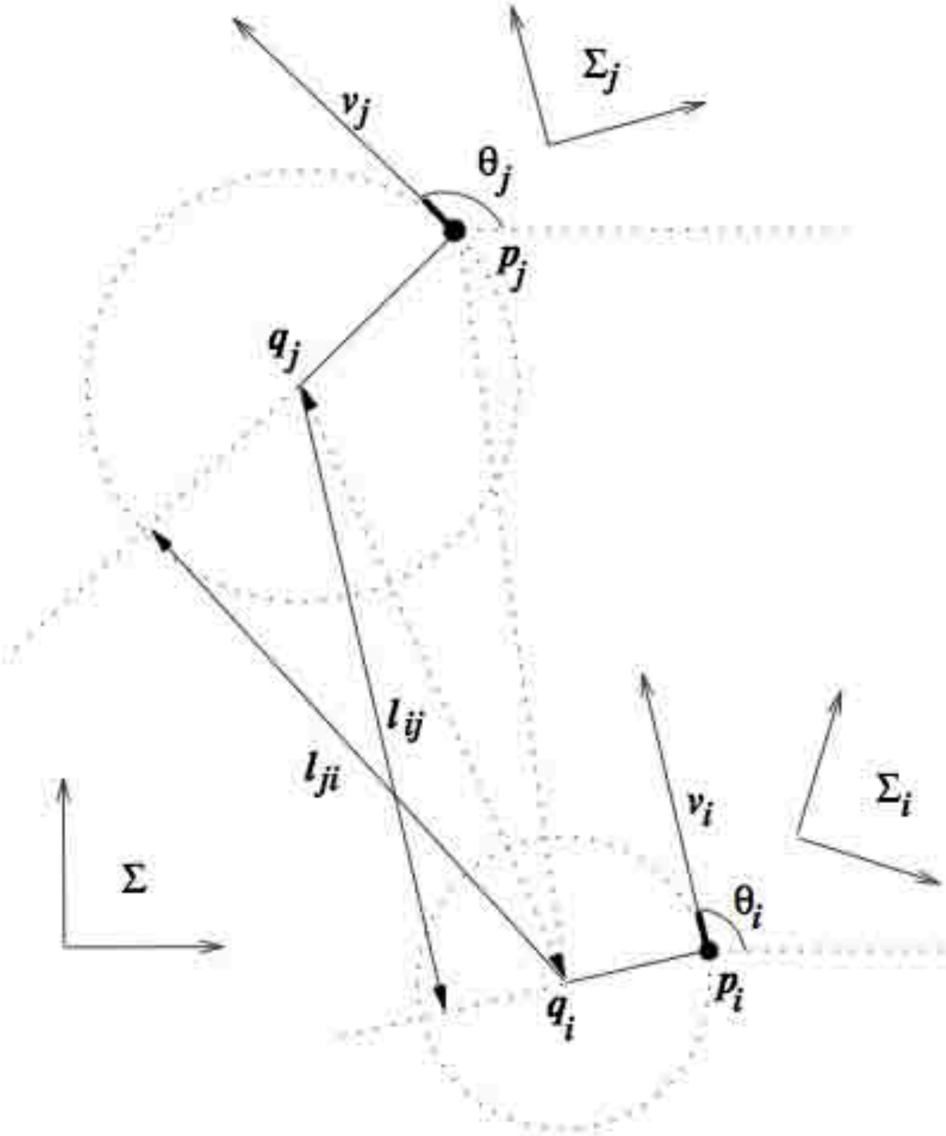
•Amigobot 多机器人系统







•纳米柔性电子制造平台（自主研发）



v_j
Σ_j
θ_j
p_j
q_j
l_{ij}
l_{ji}
v_i
Σ_i
Σ
θ_i
p_i
q_i